CIEM5000: Structural Engineering Base

The Matrix Method in Statics

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The Matrix Method

Main steps:

- Extract element matrices
- Impose nodal equilibrium
- Impose boundary conditions
- Solve for unknown displacements
- Postprocess results

This week:

- Recap differential equation for structures
- Degrees of freedom at nodes
- Local and global stiffness matrix
- Neumann and Diriclet boundary conditions
- Local-global transformations
- Example: Displacements of extension bar
- Workshop: Implement and check missing components, and solve a complicated frame

Learning Objectives

At the end of this module, you should be able to:

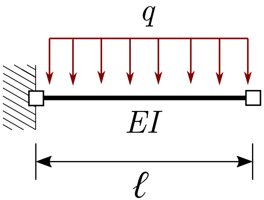
- Translate the main steps of the matrix method into a set of programming classes with distinct tasks
- Extend the classes to solve arbitrarily complex frame problems in statics
- Postprocess the analyses and recover continuum fields exactly

Learning setup:

- Lectures on theoretical aspects ($2 \times 2\,\mathrm{h}$)
- Two guided, non-graded workshops ($2 \times 2\,\mathrm{h}$), solutions provided afterwards
- Additional non-compulsory assignments exercises which you're ready for after the workshops
- Graded assignment as part of report

Recap: A single-field problem

Getting to an ODE:



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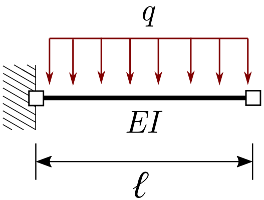
$$\varphi = -\frac{\mathrm{d}w}{\mathrm{d}x} \quad \kappa = \frac{\mathrm{d}\varphi}{\mathrm{d}x}$$



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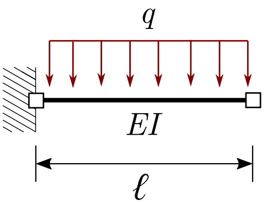
$$M = EI\kappa$$



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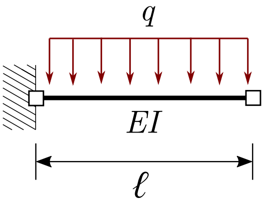
- Equilibrium relations:

$$\frac{\mathrm{d}V}{\mathrm{d}x} = -q \quad \frac{\mathrm{d}M}{\mathrm{d}x} = V$$



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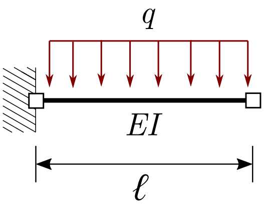

Combining it all into a single differential equation:

$$EI\frac{\mathrm{d}^4w}{\mathrm{d}x^4} = q$$

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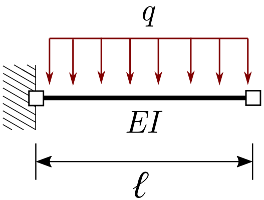
Solving the ODE (strong form!):

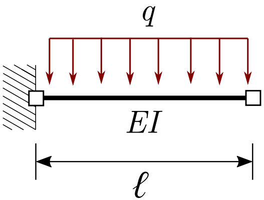


Recap: A single-field problem

Solving the ODE (strong form!):

- Integrate the ODE, exposing integration constants:

$$w(x) = \frac{qx^4}{24EI} + \frac{C_1x^3}{6} + \frac{C_2x^2}{2} + C_3x + C_4$$



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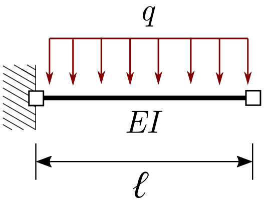
$$w(0) = 0 \quad \varphi(0) = 0 \quad M(\ell) = 0 \quad V(\ell) = 0$$



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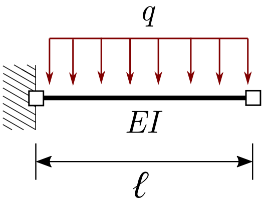
- Solve the system for the constants:

$$C_1 = -\frac{q\ell}{EI} \quad C_2 = \frac{q\ell^2}{2EI} \quad C_3 = C_4 = 0$$



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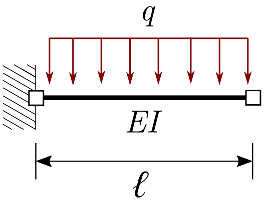
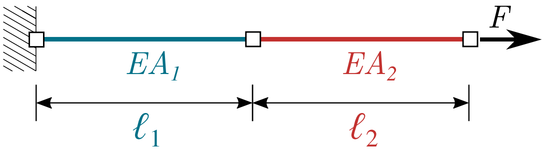


Substituting the constants, a final solution for w can be found:

$$w(x) = \frac{qx^4}{24EI} - \frac{q\ell x^3}{6EI} + \frac{q\ell^2x^2}{4EI}$$

Recap: A two-field problem



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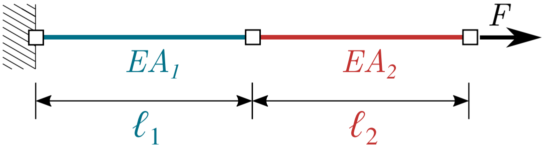

Field 1:

ODE: $EA_1 \dfrac{\mathrm{d}^2 u_1}{\mathrm{d}x^2} = 0$

Field: $u_1 = C_1 x + C_2$

BC: $u_1(0) = 0$

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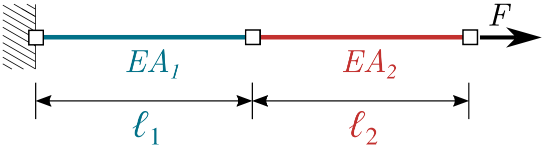

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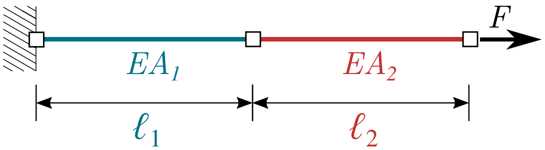

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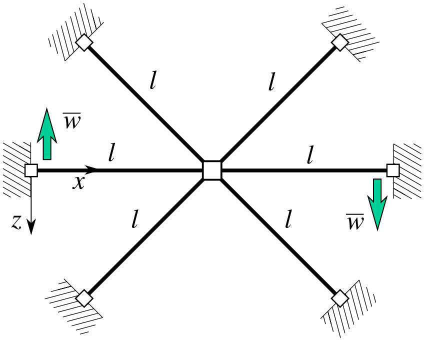
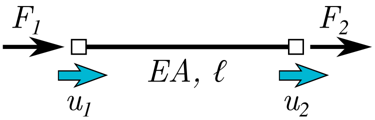
IC: $u_1(\ell_1) = u_2(\ell_1) \quad N_1(\ell_1) = N_2(\ell_1)$

Okay, easy. But how about this one?

Integration constants? Interface conditions? It gets annoying very quickly...



Is there an easier way? Deformation of a single element



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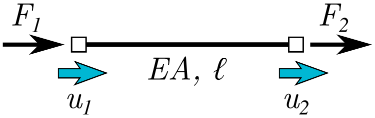

ODE solution:

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$$u(x) = C_1x + C_2$$

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$$C_1 = \frac{u_2 - u_1}{\ell} \quad C_2 = u_1$$

$$u = u_1\left(1 - \frac{x}{\ell}\right) + u_2\frac{x}{\ell}$$

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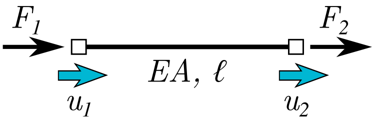

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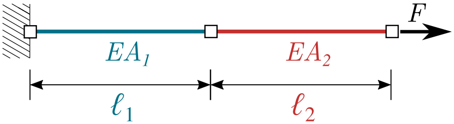
Element and node forces:

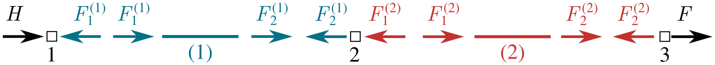
$$N = \frac{EA}{\ell}(u_2 - u_1)$$

$$F_1 = -N_1$$

$$F_2 = N_2$$

How to combine elements? Nodal equilibrium





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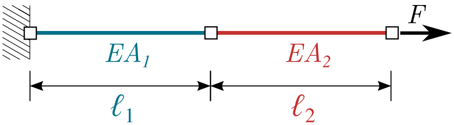

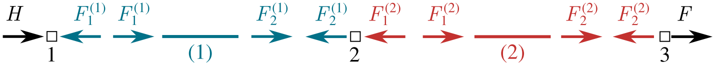

Node equilibrium:

$$\sum F_1 = 0 \Rightarrow -\frac{EA_1}{\ell_1}u_1 + \frac{EA_1}{\ell_1}u_2 + H = 0$$

$$\sum F_2 = 0 \Rightarrow \frac{EA_1}{\ell_1}u_1 - \frac{EA_1}{\ell_1}u_2 - \frac{EA_2}{\ell_2}u_2 + \frac{EA_2}{\ell_2}u_3 = 0$$

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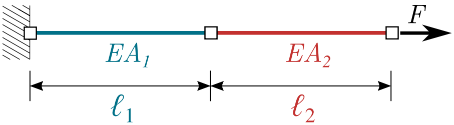

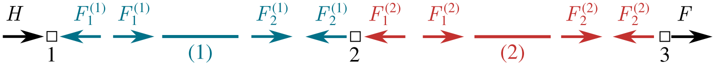

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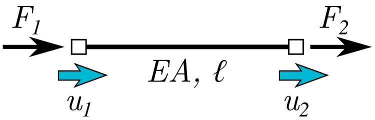
Combining and rearranging:

$$-\sum_e \mathbf{f}^e + \mathbf{f}_{\text{nodal}} = \mathbf{0}$$

$$\sum_e \mathbf{f}^e = \mathbf{f}_{\text{nodal}}$$

Deformation of a single element – matrix form



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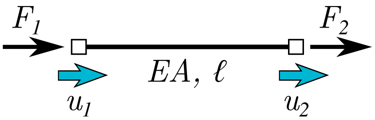

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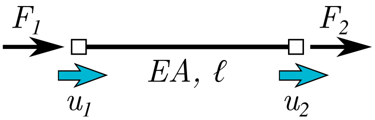

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Edge forces:

$$F_1 = -N_1$$

$$F_2 = N_2$$

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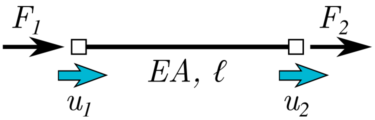

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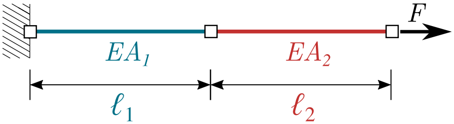
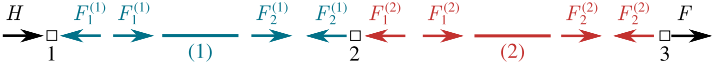
Relating $\mathbf{f}$ and $\mathbf{u}$:

$$\frac{EA}{\ell}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$\mathbf{K}^{(e)}\mathbf{u}^{(e)} = \mathbf{f}^{(e)}$$

Nodal equilibrium – matrix form





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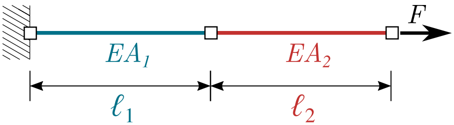

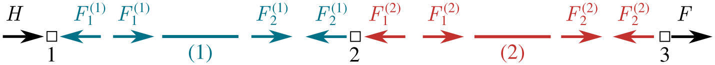

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$$\sum F_1 = 0 \Rightarrow -\frac{EA_1}{\ell_1}u_1 + \frac{EA_1}{\ell_1}u_2 + H = 0$$

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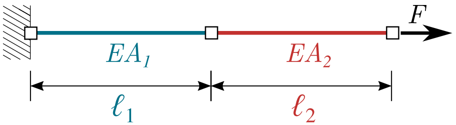

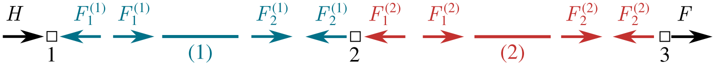

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Combining and rearranging:

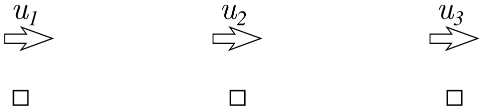
$$\begin{bmatrix} \frac{EA_1}{\ell_1} & -\frac{EA_1}{\ell_1} & 0 \\ -\frac{EA_1}{\ell_1} & \frac{EA_1}{\ell_1} + \frac{EA_2}{\ell_2} & -\frac{EA_2}{\ell_2} \\ 0 & -\frac{EA_2}{\ell_2} & \frac{EA_2}{\ell_2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} H \\ 0 \\ F \end{bmatrix}$$

$$\mathbf{K}\mathbf{u} = \mathbf{f}$$

A more structured way to work

Steps:

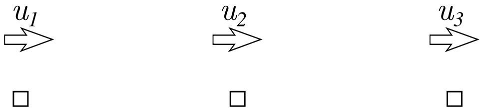
- Identify degrees of freedom at nodes (DOFs)



A more structured way to work

Steps:

- Identify degrees of freedom at nodes (DOFs)
- Initialize the system with zeros

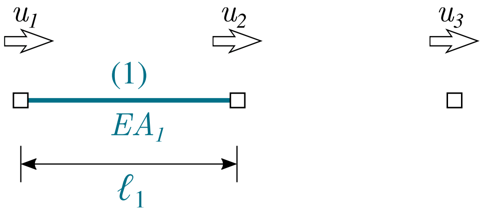


$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A more structured way to work

Steps:

- Identify degrees of freedom at nodes (DOFs)
- Initialize the system with zeros
- Assemble stiffness, element by element



$$\begin{bmatrix} \frac{EA_1}{\ell_1} & -\frac{EA_1}{\ell_1} & 0 \\ -\frac{EA_1}{\ell_1} & \frac{EA_1}{\ell_1} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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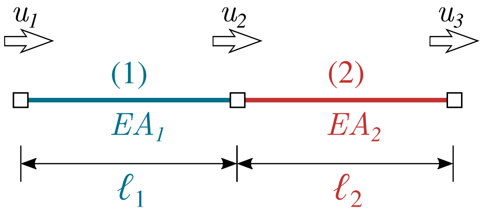
- Assemble stiffness, element by element

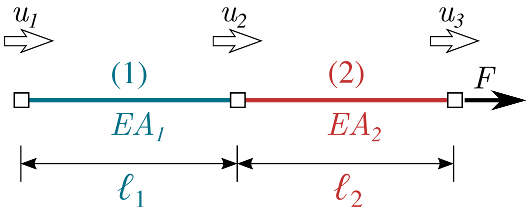


$$\begin{bmatrix} \frac{EA_1}{\ell_1} & -\frac{EA_1}{\ell_1} & 0 \\ -\frac{EA_1}{\ell_1} & \frac{EA_1}{\ell_1} + \frac{EA_2}{\ell_2} & -\frac{EA_2}{\ell_2} \\ 0 & -\frac{EA_2}{\ell_2} & \frac{EA_2}{\ell_2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A more structured way to work

Steps:

- Identify degrees of freedom at nodes (DOFs)
- Initialize the system with zeros
- Assemble stiffness, element by element
- Apply external loads (Neumann BCs)

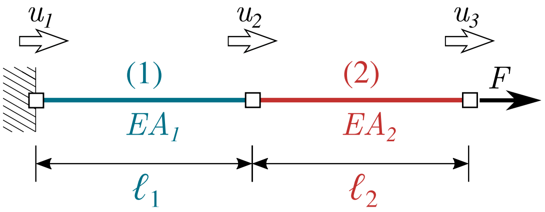


$$\begin{bmatrix} \frac{EA_1}{\ell_1} & -\frac{EA_1}{\ell_1} & 0 \\ -\frac{EA_1}{\ell_1} & \frac{EA_1}{\ell_1}+\frac{EA_2}{\ell_2} & -\frac{EA_2}{\ell_2} \\ 0 & -\frac{EA_2}{\ell_2} & \frac{EA_2}{\ell_2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ F \end{bmatrix}$$

A more structured way to work

Steps:

- Identify degrees of freedom at nodes (DOFs)
- Initialize the system with zeros
- Assemble stiffness, element by element
- Apply external loads (Neumann BCs)
- Apply prescribed displacements (Dirichlet BCs)

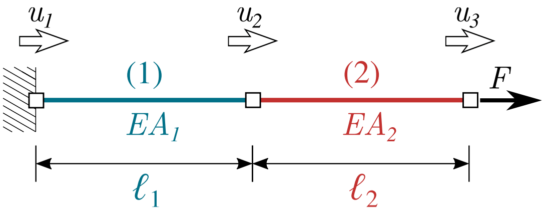


$$\begin{bmatrix} \frac{EA_1}{\ell_1} & -\frac{EA_1}{\ell_1} & 0 \\ -\frac{EA_1}{\ell_1} & \frac{EA_1}{\ell_1}+\frac{EA_2}{\ell_2} & -\frac{EA_2}{\ell_2} \\ 0 & -\frac{EA_2}{\ell_2} & \frac{EA_2}{\ell_2} \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} H \\ 0 \\ F \end{bmatrix}$$

A more structured way to work

Steps:

- Identify degrees of freedom at nodes (DOFs)
- Initialize the system with zeros
- Assemble stiffness, element by element
- Apply external loads (Neumann BCs)
- Apply prescribed displacements (Dirichlet BCs)
- Solve for the unkown nodal displacements



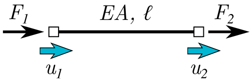
$$\begin{bmatrix} \frac{EA_1}{\ell_1} & -\frac{EA_1}{\ell_1} & 0 \\ -\frac{EA_1}{\ell_1} & \frac{EA_1}{\ell_1} + \frac{EA_2}{\ell_2} & -\frac{EA_2}{\ell_2} \\ 0 & -\frac{EA_2}{\ell_2} & \frac{EA_2}{\ell_2} \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} H \\ 0 \\ F \end{bmatrix}$$

$$\begin{bmatrix} \frac{EA_1}{\ell_1} + \frac{EA_2}{\ell_2} & -\frac{EA_2}{\ell_2} \\ -\frac{EA_2}{\ell_2} & \frac{EA_2}{\ell_2} \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ F \end{bmatrix}$$

$$u_2 = \frac{F\ell_1}{EA_1} \qquad u_3 = \frac{F\left(EA_1\ell_2 + EA_2\ell_1\right)}{EA_1\, EA_2}$$

Other element types

Different element kinematics and stiffness matrices, same procedure



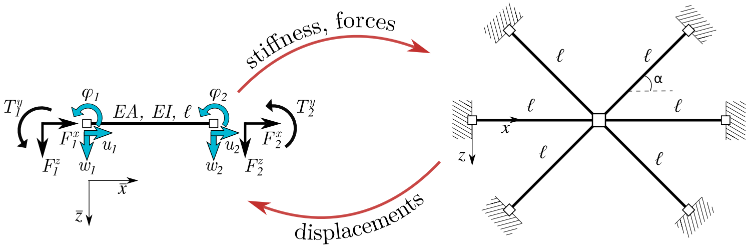
$$\mathbf{K}^{(e)} = \begin{bmatrix} \frac{EA}{\ell} & -\frac{EA}{\ell} \\ -\frac{EA}{\ell} & \frac{EA}{\ell} \end{bmatrix}$$

$$\begin{bmatrix} \frac{EA}{\ell} & 0 & 0 & -\frac{EA}{\ell} & 0 & 0 \\ 0 & \frac{12EI}{\ell^3} & -\frac{6EI}{\ell^2} & 0 & -\frac{12EI}{\ell^3} & -\frac{6EI}{\ell^2} \\ 0 & -\frac{6EI}{\ell^2} & \frac{4EI}{\ell} & 0 & \frac{6EI}{\ell^2} & \frac{2EI}{\ell} \\ -\frac{EA}{\ell} & 0 & 0 & \frac{EA}{\ell} & 0 & 0 \\ 0 & -\frac{12EI}{\ell^3} & \frac{6EI}{\ell^2} & 0 & \frac{12EI}{\ell^3} & \frac{6EI}{\ell^2} \\ 0 & -\frac{6EI}{\ell^2} & \frac{2EI}{\ell} & 0 & \frac{6EI}{\ell^2} & \frac{4EI}{\ell} \end{bmatrix}$$

Element orientations, local-global transformations

Defining a local (element) coordinate system is useful:

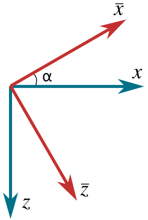
- Single stiffness matrix for every element!
- Assembly: From local to global
- Postprocessing: From global to local



Local-global transformations

Transformations for an arbitrary vector:

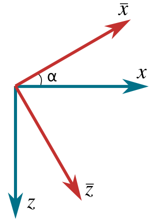
$$\begin{bmatrix} v_{\bar{x}} \\ v_{\bar{z}} \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} v_x \\ v_z \end{bmatrix} \qquad \begin{bmatrix} v_x \\ v_z \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}}_{\mathbf{R}^\mathrm{T}} \begin{bmatrix} v_{\bar{x}} \\ v_{\bar{z}} \end{bmatrix}$$



Local-global transformations

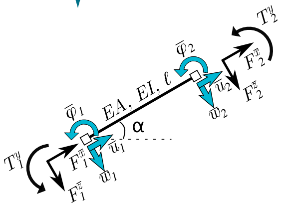
Transformations for an arbitrary vector:

$$\begin{bmatrix} v_{\bar{x}} \\ v_{\bar{z}} \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} v_x \\ v_z \end{bmatrix} \qquad \begin{bmatrix} v_x \\ v_z \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}}_{\mathbf{R}^\mathrm{T}} \begin{bmatrix} v_{\bar{x}} \\ v_{\bar{z}} \end{bmatrix}$$

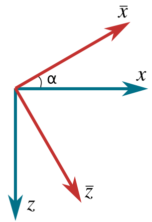


Transformations for a complete element:

$$\begin{bmatrix} \bar{u}_1 \\ \bar{w}_1 \\ \bar{\varphi}_1 \\ \bar{u}_2 \\ \bar{w}_2 \\ \bar{\varphi}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & 0 & 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{T}} \begin{bmatrix} u_1 \\ w_1 \\ \varphi_1 \\ u_2 \\ w_2 \\ \varphi_2 \end{bmatrix}$$



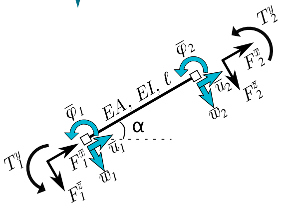
Local-global transformations



Transformations for an arbitrary vector:

$$\begin{bmatrix} v_{\bar{x}} \\ v_{\bar{z}} \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} v_x \\ v_z \end{bmatrix} \qquad \begin{bmatrix} v_x \\ v_z \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}}_{\mathbf{R}^\mathrm{T}} \begin{bmatrix} v_{\bar{x}} \\ v_{\bar{z}} \end{bmatrix}$$

Transformations for a complete element:



$$\begin{bmatrix} \bar{u}_1 \\ \bar{w}_1 \\ \bar{\varphi}_1 \\ \bar{u}_2 \\ \bar{w}_2 \\ \bar{\varphi}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & 0 & 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{T}} \begin{bmatrix} u_1 \\ w_1 \\ \varphi_1 \\ u_2 \\ w_2 \\ \varphi_2 \end{bmatrix}$$

With this we can define the following important transformations:

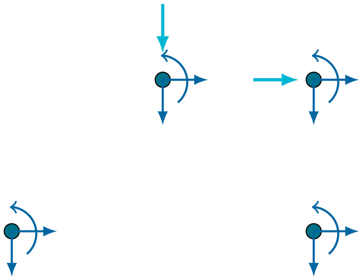
$$\bar{\mathbf{u}} = \mathbf{T}\mathbf{u} \quad \bar{\mathbf{f}} = \mathbf{T}\mathbf{f} \quad \mathbf{u} = \mathbf{T}^\mathrm{T}\bar{\mathbf{u}} \quad \mathbf{f} = \mathbf{T}^\mathrm{T}\bar{\mathbf{f}}$$

$$\mathbf{K} = \mathbf{T}^\mathrm{T}\bar{\mathbf{K}}\mathbf{T}$$

Coding the matrix method

The method is well structured and can be broken down as follows:

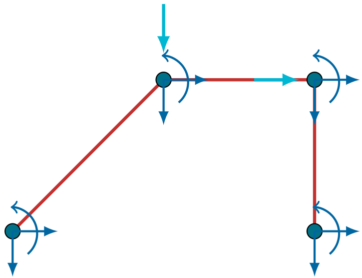
- A list of Nodes floating in space with loads and DOFs associated to them



Coding the matrix method

The method is well structured and can be broken down as follows:

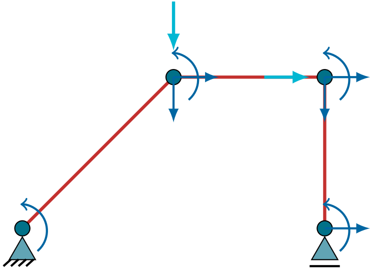
- A list of Nodes floating in space with loads and DOFs associated to them
- A list of Elements defined by linking two nodes together



Coding the matrix method

The method is well structured and can be broken down as follows:

- A list of Nodes floating in space with loads and DOFs associated to them
- A list of Elements defined by linking two nodes together
- A Constrainer to apply Dirichlet boundary conditions

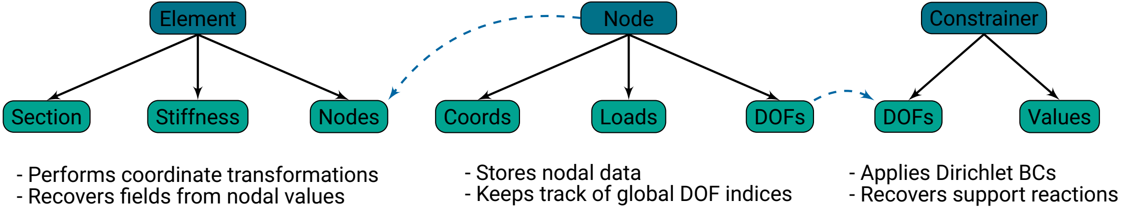


Coding the matrix method

The method is well structured and can be broken down as follows:

- A list of Nodes floating in space with loads and DOFs associated to them
- A list of Elements defined by linking two nodes together
- A Constrainer to apply Dirichlet boundary conditions

With this in mind, we can define object-oriented code which can be loaded as a python package:



Outlook

First ungraded workshop:

- Get familiar with an initial Python code
- Implement a few missing parts and perform some sanity checks
- Apply your implementations to a small structure
- Have Git, Anaconda and Jupyter installed and ready
- Never used Git? Let me (Tom) know!

Next week:

- One more lecture on theoretical aspects
- Second ungraded workshop to add more implementations and solve a more advanced structure
- Graded assignment: Implement, check and apply new features required for complicated frame structure and additional results.